

Multiple-Choice Questions

Please choose only one answer and insert in PENCIL in your Scantron sheet.

1. [3 marks] Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$.

- (a) -1
- (b) $1/2$
- (c) $3/2$
- (d) 0

2. [4 marks] Let $f(x) = (\ln x)^x$. Evaluate $f'(e)$, where $e = 2.718 \dots$ is Euler's number. In other words, find the derivative of f at $x = e$.

- (a) $f'(e) = 0$
- (b) $f'(e) = 1$
- (c) $f'(e) = -1$
- (d) $f'(e) = 2$

3. [3 marks] Let $f(x) = 3|x - 1|$. Calculate

$$L = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}.$$

- (a) $L = 0$
- (b) $L = 1$
- (c) $L = -1$
- (d) This limit does not exist

4. [4 marks] Find the derivative of the function f defined by $f(x) = \sqrt{1 + (\ln x)^2}$.

- (a) $\frac{\ln x}{x \sqrt{1 + (\ln x)^2}}$
- (b) $\frac{1}{2 \sqrt{1 + (\ln x)^2}}$
- (c) $\frac{1}{2x \sqrt{1 + (\ln x)^2}}$
- (d) $\frac{\ln x}{2x^2 \sqrt{1 + (\ln x)^2}}$

5. [3 marks] Let $f(x) = 3^x x^3$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.
- (a) $9 + \ln 27$
 - (b) $2 \ln 3$
 - (c) $1 + \ln 3$
 - (d) 0
6. [3 marks] A differentiable function f with a differentiable inverse, F , has the property that $f'(1) = -2$ and $F(0) = 1$. What is the value of the derivative of the inverse of f at $x = 0$? That is, calculate $F'(0)$.
- (a) 2
 - (b) $-1/2$
 - (c) 0
 - (d) $1/2$
7. [3 marks] Find the derivative of the function f defined by $f(x) = \text{Arcsin}(\frac{2}{x})$ for $x > 0$.
- (a) $f'(x) = -(\frac{2}{x^2})\text{Arccos}(\frac{2}{x})$
 - (b) $f'(x) = \sqrt{4 - x^2}$
 - (c) $f'(x) = -\frac{2}{x\sqrt{x^2 - 4}}$
 - (d) $f'(x) = \frac{1}{x\sqrt{x^2 - 4}}$
8. [4 marks] Evaluate the limit: $L = \lim_{x \rightarrow \infty} x \sin\left(\frac{2}{x}\right)$.
- (a) $L = 1/2$
 - (b) $L = 0$
 - (c) $L = 2$
 - (d) $L = -1$
9. [4 marks] Let y be given implicitly as a differentiable function of x by $y = \cos(x + y)$. Calculate the value of the derivative $\frac{dy}{dx}$ at the point (x, y) where $x = \pi/2$, $y = 0$:
- (a) $1/3$,
 - (b) -1 ,
 - (c) 0 ,
 - (d) $-1/2$

10. [4 marks] Evaluate $L = \lim_{x \rightarrow \infty} \frac{2^x - 1}{3^x - 1}$ using any method.

- (a) $L = 0$
- (b) $L = 1$
- (c) $L = 2$
- (d) $L = -1$

11. [4 marks] Evaluate

$$I = \lim_{t \rightarrow 0^+} \frac{d}{dt} \int_1^{\sqrt{t}} \frac{\sin x^2}{x} dx$$

- (a) $I = 0$
- (b) $I = \frac{1}{2}$
- (c) This limit does not exist
- (d) $I = \frac{5}{2}$

12. [4 marks] The function f defined by $f(x) = 2x^3 + 3x^2 - 12x + 1$ is decreasing at each point of which of the following intervals?

- (a) $0 < x < 7$
- (b) $-6 < x < -4$
- (c) $3 < x < 5$
- (d) $-2 < x < 1$

13. [4 marks] For what values of a and b does the function f defined by

$$f(x) = a \ln x + bx^2 + x$$

have the two critical points, $x = 1$ and $x = 2$?

- (a) $a = 0, b = 1$
- (b) $a = -2/3, b = -1/6$
- (c) $a = -1/2, b = 1/3$
- (d) $a = 1, b = -1/2$

14. [4 marks] Find all the points of inflection of the function f defined by $f(x) = (x + 1)^4 + e^{-x}$.

- (a) $x = 0$ only.
- (b) $x = e$ and $x = -1$ only.
- (c) $x = 0$ and $x = 1$ only.
- (d) There are no points of inflection.

15. [4 marks] Count the **number of points of inflection** of the graph of the function defined by

$$f(x) = \frac{x + 1}{x^2 + 1}.$$

- (a) 3
- (b) 1
- (c) 4
- (d) 2

16. [4 marks] Determine the interval in which the graph of the function f defined by $f(x) = e^{-x^2}$ is concave up.

- (a) $x > \sqrt{2}/2$
- (b) $x < 0$
- (c) $x < \sqrt{2}/2$
- (d) This graph is never concave up.

17. [4 marks] Find ALL the horizontal asymptotes of the graph of the function f defined by

$$f(x) = \frac{4x + \sin x}{2x + \cos x}.$$

- (a) $y = -1$, $y = 1$ only
- (b) $y = 0$, $y = 1$ only
- (c) $y = 2$ only
- (d) $y = 3$ only

18. [4 marks] An antiderivative of $f(x) = \cot(2x + 1)$ is given by

- (a) $\ln(\sec(2x + 1)) - 1$
- (b) $\frac{1}{2} \ln |\cos(2x + 1)| + 2$
- (c) $\frac{1}{2} \ln |\sin(2x + 1)|$
- (d) $\frac{1}{2} \ln |\csc^2(2x + 1)|$

19. [4 marks] Evaluate $\int_0^1 x e^{-x} dx$

- (a) 1
- (b) $1 - \frac{2}{e}$
- (c) $\frac{e^2 - 1}{2}$
- (d) $\frac{e - 1}{2}$

20. [4 marks] Evaluate $\int t^2 \ln 3t dt$.

- (a) $\frac{1}{3} t^3 \ln(3t) - \frac{t^3}{9} + C$
- (b) $\frac{1}{3} t^3 \ln t - \frac{t}{9} + C$
- (c) $3(\ln 3t)^2 + \frac{1}{9} + C$
- (d) $2t^2(\ln 3t) + \frac{t}{9} + C$

21. [4 marks] The improper integral $\int_0^\infty x^2 2^{-x^3} dx$ has the value

- (a) $-\frac{1}{2 \ln 3}$
- (b) $\frac{1}{2}$
- (c) $\frac{1}{9}$
- (d) $\frac{1}{3 \ln 2}$

22. [4 marks] Evaluate and simplify the indefinite integral: $\int x^2 e^{3x} dx$.

(a) $e^{3x} \left(\frac{2}{27} - \frac{2x}{9} + \frac{x^2}{3} \right) + C$

(b) $e^{3x} \left(\frac{1}{27} - \frac{2x}{9} + \frac{x^2}{27} \right) + C$

(c) $e^{-3x} \left(\frac{2}{9} - \frac{2x}{27} + \frac{x^2}{6} \right) + C$

(d) $e^{3x} \left(\frac{1}{27} - \frac{2x}{27} + \frac{x^2}{9} \right) + C$

23. [4 marks] Evaluate

$$\int \frac{x dx}{(x+1)(x-2)}$$

using the method of partial fractions.

(a) $\frac{2}{3} \ln|x+1| + \frac{4}{3} \ln|x-2| + C$

(b) $\frac{1}{3} \ln|x+1| - \frac{2}{3} \ln|x-2| + C$

(c) $\frac{1}{3} \ln|x+1| + \frac{2}{3} \ln|x-2| + C$

(d) $\frac{1}{2} \ln|x+1| + \frac{1}{3} \ln|x-2| + C$

24. [4 marks] Evaluate the indefinite trigonometric integral

$$\int \sin^2 x \cos^3 x dx.$$

(a) $\frac{\sin^3 x \cos^3 x}{3} + C$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

(c) $\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$

(d) $\frac{\sin^3 x}{5} - \frac{\sin^5 x}{3} + C$

25. [4 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant and between the curves defined by $y = \sqrt{x}$ and $y = x^2$ about the y -axis.

(a) $\int_0^1 \pi(x^2 - \sqrt{x}) \, dx$

(b) $\int_0^1 \sqrt{x} \, dx$

(c) $\int_0^1 2\pi x (\sqrt{x} - x^2) \, dx$

(d) $\int_0^1 \pi (\sqrt{x} - x^2) \, dx$

26. [5 marks] Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{x^2}{y^3}$$

subject to the initial condition $y(0) = 1$. Then its value at the point $x = 1$, that is, $y(1)$ is given approximately by

(a) $y(1) = \sqrt[4]{\frac{7}{3}} \approx 1.236$

(b) $y(1) = 0$

(c) $y(1) = \sqrt[4]{\frac{1}{3}} \approx 0.760$

(d) $y(1) = \sqrt[4]{\frac{5}{12}} \approx 0.803$

END